Consistent interactions for high-spin fermion fields

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1. Introduction

The Rarita-Schwinger formalism

Rarita-Schwinger equations

$$(i\partial\!\!\!/ - m)\psi_{\mu_1\ldots\mu_n} = 0$$
$$\gamma^{\mu_1}\psi_{\mu_1\mu_2\ldots\mu_n} = 0$$

- $\psi_{\mu_1\ldots\mu_n}$: spin-$(n+\frac{1}{2})$ Rarita-Schwinger (R-S) field
- Unphysical components are eliminated through R-S constraints
- Massless theory requires invariance under R-S gauge

$$\psi_{\mu_1\ldots\mu_n} \to \psi_{\mu_1\ldots\mu_n} + \frac{i}{n(n-1)!}\sum_{P(\mu)}\partial_{\mu_1}\xi_{\mu_2\ldots\mu_n}$$

$$\gamma^{\mu_1}\xi_{\mu_1\mu_2\ldots\mu_{n-1}} = 0$$

Interacting Rarita-Schwinger fields

On-shell case

- On-shell R-S field is described by R-S spinor
- Unphysical components of R-S spinor decouple from the interaction

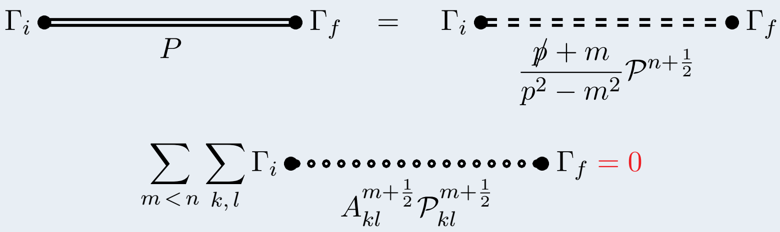
Off-shell case

- Off-shell R-S field is described by R-S propagator
- Unphysical components of R-S propagator do not decouple a priori
- Consistent interaction should be invariant under certain **local gauge**

2. Consistent interactions

Consistency and locality of the interaction

Consistent interaction for $\psi^*_{\mu_1 \dots \mu_n}$



Interpretation

- Interaction is mediated purely by physical component of R-S field

Consistency and locality of the interaction

Preservation of locality

- $\mathcal{P}^{n+\frac{1}{2}}_{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(p)$ contains p^{-2k} singularities $(k \in \{1,\ldots,n\})$
  - ▶ Transversality conditions of interaction vertices

$$p_{\mu_k}\Gamma_I^{\mu_1\ldots\mu_n} = 0 \qquad k \in \{1,\ldots,n\}$$

  - ▶ Equivalent to invariance of interaction under unconstrained R-S gauge

$$\psi_{\mu_1\ldots\mu_n} \rightarrow \psi_{\mu_1\ldots\mu_n} + \frac{i}{n(n-1)!}\sum_{P(\mu)}\partial_{\mu_1}\chi_{\mu_2\ldots\mu_n}$$

- Unconstrained R-S gauge concurrently guarantees consistent interaction

The gauge-invariant Rarita-Schwinger field

Gauge-invariant Rarita-Schwinger field

$$G_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n} = O^{n+\frac{1}{2}}_{(\mu_1\ldots\mu_n,\nu_1\ldots\nu_n)\lambda_1\ldots\lambda_n}(\partial)\psi^{\lambda_1\ldots\lambda_n}$$

- Spin-$(n+\frac{1}{2})$ interaction operator

$$O^{n+\frac{1}{2}}_{(\mu_1\ldots\mu_n,\nu_1\ldots\nu_n)\lambda_1\ldots\lambda_n}(\partial) = \frac{1}{(n!)^2}\sum_{P(\nu)}\sum_{P(\lambda)} O^{\frac{3}{2}}_{(\mu_1,\nu_1)\lambda_1}(\partial)\cdots O^{\frac{3}{2}}_{(\mu_n,\nu_n)\lambda_n}(\partial)$$

- Pascalutsa's spin-$\frac{3}{2}$ interaction operator

$$O^{\frac{3}{2}}_{(\mu,\nu)\lambda}(\partial) = i\left(\partial_\mu g_{\nu\lambda} - \partial_\nu g_{\mu\lambda}\right)$$

The gauge-invariant Rarita-Schwinger field

Properties

- $G_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n}$ totally symmetric in μ and ν indices, and

$$G_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n} = (-1)^n G_{\nu_1\ldots\nu_n,\mu_1\ldots\mu_n}$$

- Invariance under unconstrained R-S gauge assured by

$$\partial^{\lambda_k} O^{n+\frac{1}{2}}_{(\mu_1\ldots\mu_n,\nu_1\ldots\nu_n)\lambda_1\ldots\lambda_n}(\partial) = 0$$

Construction of consistent interaction theory

$$\mathcal{L}_I = \overline{G}_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n} T^{\mu_1\ldots\mu_n\nu_1\ldots\nu_n} + \overline{T}_{\mu_1\ldots\mu_n\nu_1\ldots\nu_n} G^{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n}$$

Consistent interaction theories

Reduced gauge-invariant Rarita-Schwinger field

$$\Psi_{\mu_1\ldots\mu_n} = \gamma^{\nu_1}\cdots\gamma^{\nu_n} G_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n}$$

Properties

- $\Psi_{\mu_1\ldots\mu_n}$ obeys R-S constraints

$$\partial^{\mu_1}\Psi_{\mu_1\ldots\mu_n} \to 0 \qquad \text{and} \qquad \gamma^{\mu_1}\Psi_{\mu_1\ldots\mu_n} \to 0$$

- Intuitive interaction structure

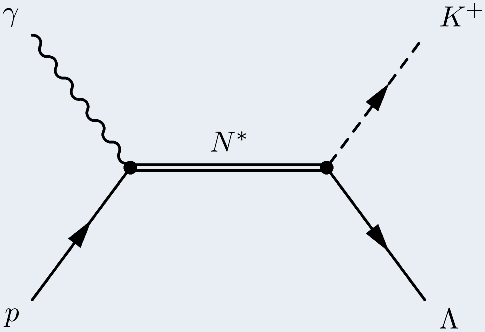
$$\mathcal{O}^{n+\frac{1}{2}} \cdot P \cdot \mathcal{O}^{n+\frac{1}{2}} = p^{2n}\frac{\not{p}+m}{p^2-m^2}\mathcal{P}^{n+\frac{1}{2}}$$

3. Consistent interactions in hadron physics

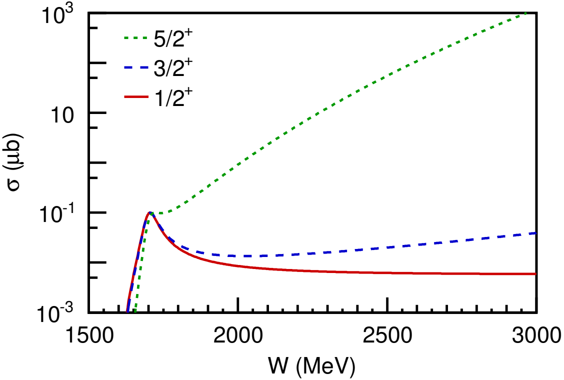
Inconsistency of standard hadronic form factors

Practical use of consistent interactions: an example

- Isobar modeling of s channel of $K^+\Lambda$ photoproduction
- High-spin nucleon resonances are created from the proton



Inconsistency of standard hadronic form factors



Toy N^*

$$m_R = 1700 \text{ MeV}$$
$$\Gamma_R = 50 \text{ MeV}$$
$$J_R^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$$

Hadronic form factor required to suppress high-energy behavior

Inconsistency of standard hadronic form factors

Standard hadronic form factors

- Dipole form

$$F_d(s; m_R, \Lambda_R) = \frac{\Lambda_R^4}{(s - m_R^2)^2 + \Lambda_R^4}$$

- Gaussian form

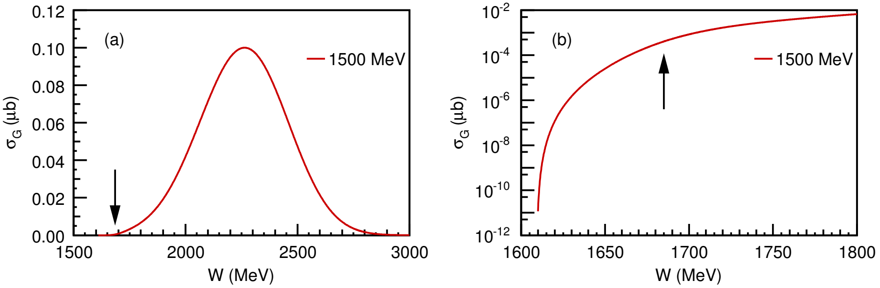
$$F_G(s; m_R, \Lambda_R) = \exp\left(-\frac{(s - m_R^2)^2}{\Lambda_R^4}\right)$$

- Specific s channel contribution to $p(\gamma, K^+)\Lambda$:

$$N^* = N(1680)\ F_{15} \begin{cases} m_R = 1685 \text{ MeV} \\ \Gamma_R = 130 \text{ MeV} \\ J_R^P = \frac{5}{2}^+ \end{cases}$$

Inconsistency of standard hadronic form factors

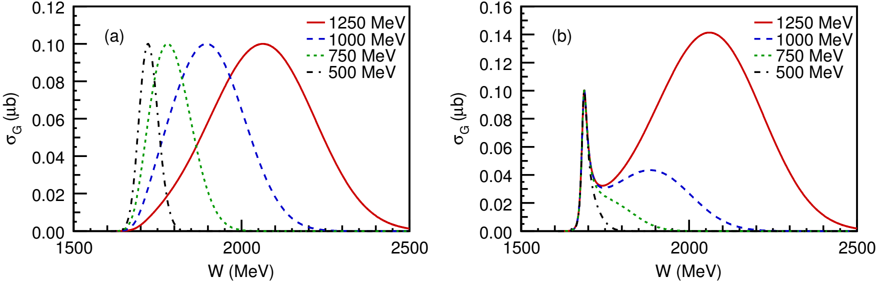
$$\gamma p \to N(1680)\ F_{15} \to K^+\Lambda, \qquad F = F_G$$



Remarks

- Artificial bump arises when σ is cut off by F_G
- No resonance peak is observed at $W \approx m_R$

Inconsistency of standard hadronic form factors



Remarks

- Lowering Λ_R results in shift of artificial bump towards W_0
- Lowering Λ_R only effective when Γ_R is "small"
  - practically all N^*'s listed by PDG have "large" Γ_R

The multidipole-Gauss form factor

Cause of unphysical behavior

- Rise of σ with W and fall of F_G with W combines to form artificial bump
- Fast increase of σ with W causes resonance peak with "large" Γ_R to vanish

Remedy

- High-energy behavior of σ

$$\sigma(s) \propto (s^2)^{J_R - \frac{1}{2}}$$

needs to be regulated

The multidipole-Gauss form factor

Multidipole-Gauss form factor

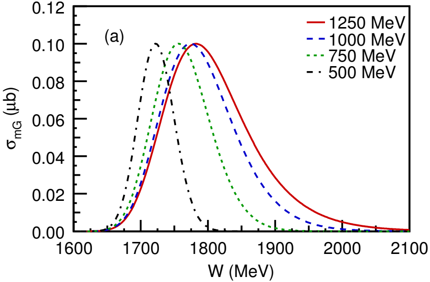
$$F_{mG}(s; m_R, \Lambda_R, \Gamma_R, J_R) =$$

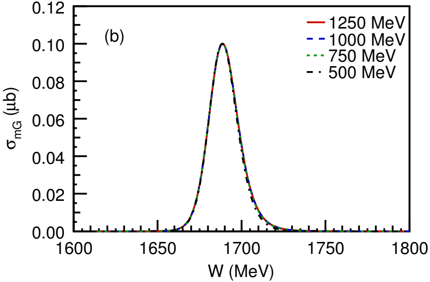
$$\left(\frac{m_R^2 \widetilde{\Gamma}_R^2(J_R)}{(s - m_R^2)^2 + m_R^2 \widetilde{\Gamma}_R^2(J_R)} \right)^{J_R - \frac{1}{2}} \exp\left(-\frac{(s - m_R^2)^2}{\Lambda_R^4} \right)$$

- Dipole part of F_{mG} raises multiplicity of propagator pole
- Modified decay width

$$\widetilde{\Gamma}_R(J_R) = \frac{\Gamma_R}{\sqrt{2^{\frac{1}{2J_R}} - 1}}$$

The multidipole-Gauss form factor





Remarks

- Artificial bump is removed and resonance peak is restored
- Threshold effects for $m_R - \frac{\Gamma_R}{2} \approx W_0$
  - Peak position not at $W = m_R$
  - Peak position and width are function of Λ_R

Conclusions

Summary

- Invariance under unconstrained Rarita-Schwinger gauge guarantees consistency of the interaction
- Standard hadronic form factors give rise to unphysical behavior in energy dependence of cross section
- Multidipole-Gauss hadronic form factor leads to physically acceptable energy dependence of cross section

Backup slides

Backup slide I(a)

Spin-$(n+\frac{1}{2})$ projection operator

$$\mathcal{P}^{n+\frac{1}{2}}_{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(p) =$$

$$\sum_{P(\mu)}\sum_{P(\nu)}\left(\sum_{k=0}^{k_{\max,1}} \mathcal{A}^n_k \mathcal{P}_{\mu_1\mu_2}\mathcal{P}_{\nu_1\nu_2}\cdots\mathcal{P}_{\mu_{2k-1}\mu_{2k}}\mathcal{P}_{\nu_{2k-1}\nu_{2k}} \prod_{i=2k+1}^{n} \mathcal{P}_{\mu_i\nu_i}\right.$$

$$\left.+ \not{\mathcal{P}}_{\mu_1}\not{\mathcal{P}}_{\nu_1}\sum_{k=0}^{k_{\max,2}} \mathcal{B}^n_k \mathcal{P}_{\mu_2\mu_3}\mathcal{P}_{\nu_2\nu_3}\cdots\mathcal{P}_{\mu_{2k}\mu_{2k+1}}\mathcal{P}_{\nu_{2k}\nu_{2k+1}} \prod_{i=2k+2}^{n} \mathcal{P}_{\mu_i\nu_i}\right)$$

$$\mathcal{P}_{\mu\nu} = g_{\mu\nu} - \underbrace{\frac{1}{p^2}p_\mu p_\nu}_{\text{singularity}} \qquad \text{and} \qquad \not{\mathcal{P}}_\mu = \gamma^\nu \mathcal{P}_{\mu\nu} = \gamma_\mu - \underbrace{\frac{\not{p}}{p^2}p_\mu}_{\text{singularity}}$$

Backup slide I(b)

Spin-$(n+\frac{1}{2})$ propagator

$$P_{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(p) = \frac{\not{p}+m}{p^2-m^2}\widetilde{\mathcal{P}}^{n+\frac{1}{2}}_{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(p)$$

$$\begin{aligned}\mathcal{P}_{\mu\nu} &= g_{\mu\nu} - \frac{1}{p^2}p_\mu p_\nu \\ &\to g_{\mu\nu} - \frac{1}{m^2}p_\mu p_\nu \\ \not{\mathcal{P}}_\mu \not{\mathcal{P}}_\nu &= \gamma_\mu\gamma_\nu + \frac{\not{p}}{p^2}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{1}{p^2}p_\mu p_\nu \\ &\to \gamma_\mu\gamma_\nu + \frac{1}{m}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{1}{m^2}p_\mu p_\nu\end{aligned}$$

Substitutions have no effect since $p_{\mu_k}\Gamma_I^{\mu_1\ldots\mu_n} = 0$

Backup slide II(a)

Inconsistent spin-$\frac{3}{2}$ interaction theories

$$\mathcal{L}'_{\phi\psi\psi^*_\mu} = \frac{ig_0}{m_\phi}\overline{\psi}^\mu \Theta_{\mu\nu}(z_0)\Gamma\psi\partial^\nu\phi$$

$$\mathcal{L}'^{(1)}_{A_\mu\psi\psi^*_\mu} = \frac{ig_1}{2m_\psi}\overline{\psi}^\mu \Theta_{\mu\nu}(z_1)\Gamma\gamma_\lambda\psi F^{\lambda\nu}$$

$$\mathcal{L}'^{(2)}_{A_\mu\psi\psi^*_\mu} = -\frac{g_2}{4m_\psi^2}\overline{\psi}^\mu \Theta_{\mu\nu}(z_2)\Gamma\partial_\lambda\psi F^{\lambda\nu}$$

$$\mathcal{L}'^{(3)}_{A_\mu\psi\psi^*_\mu} = -\frac{g_3}{4m_\psi^2}\overline{\psi}^\mu \Theta_{\mu\nu}(z_3)\Gamma\psi\partial_\lambda F^{\lambda\nu}$$

Backup slide II(b)

Consistent spin-$(n+\frac{1}{2})$ interaction theories

$$\mathcal{L}_{\phi\psi\psi^*_{\mu_1\ldots\mu_n}} = \frac{i^n g_0}{m_\phi^{2n}} \overline{\Psi}_{\mu_1\ldots\mu_n} \Gamma \psi \partial^{\mu_1} \cdots \partial^{\mu_n} \phi$$

$$\mathcal{L}^{(1)}_{A_\mu\psi\psi^*_{\mu_1\ldots\mu_n}} = \frac{i^n g_1}{(2m_\psi)^{2n}} \overline{\Psi}_{\mu_1\ldots\mu_{n-1}\mu_n} \Gamma \gamma_\nu \partial^{\mu_1} \cdots \partial^{\mu_{n-1}} \psi F^{\nu\mu_n}$$

$$\mathcal{L}^{(2)}_{A_\mu\psi\psi^*_{\mu_1\ldots\mu_n}} = \frac{i^{n+1} g_2}{(2m_\psi)^{2n+1}} \overline{\Psi}_{\mu_1\ldots\mu_{n-1}\mu_n} \Gamma \partial_\nu \partial^{\mu_1} \cdots \partial^{\mu_{n-1}} \psi F^{\nu\mu_n}$$

$$\mathcal{L}^{(3)}_{A_\mu\psi\psi^*_{\mu_1\ldots\mu_n}} = \frac{i^{n+1} g_3}{(2m_\psi)^{2n+1}} \overline{\Psi}_{\mu_1\ldots\mu_{n-1}\mu_n} \Gamma \partial^{\mu_1} \cdots \partial^{\mu_{n-1}} \psi \partial_\nu F^{\nu\mu_n}$$